Exam Symmetry in Physics

Date	February 4, 2011
Room	X 5118.-156
Time	9:00 - 12:00
Lecturer	D. Boer

- Write your name and student number on every separate sheet of paper
- You are not allowed to use the lecture notes, nor other notes or books
- All subquestions (a, b, etc) of the three exercises have equal weight
- Answers may be given in Dutch
- Illegible handwriting will be graded as incorrect
- Good luck!

Exercise 1

Consider the dihedral group D_3: $\text{gp}\{c, b\}$ with $c^3 = b^2 = (bc)^2 = e$.

(a) Show that the cyclic group C_3 forms an invariant subgroup of D_3.

(b) Show that $D_3/C_3 \cong C_2$.

(c) Construct the character tables of C_2, C_3, and D_3.

(d) Show which irreps of C_2 can be lifted to irreps of D_3 through $D^G(g) := D^{G/N}(gN)$.

(e) Determine the Clebsch-Gordan series of the direct product rep $D^{(3)} \otimes D^{(3)}$ of D_3, where $D^{(3)}$ denotes the two-dimensional irrep of D_3.

Consider a molecule with D_3 symmetry. By application of an external magnetic field the symmetry is broken to a C_3 symmetry.

(f) Decompose the irreps of D_3 into those of C_3 using the character tables.

(g) Explain what the implications of the symmetry breaking are for the degeneracy of the energy levels of the molecule.

Exercise 2

(a) Explain what the concept of symmetry means in physics.

(b) Explain the role of representations in physics.

(c) Under which symmetry transformations is the Hamiltonian $H = \vec{p}^{\,2}/2m + V(|\vec{r}|)$ invariant?

(d) Show that $[H, U(g)] = 0$ implies that transformed states $U(g)\psi$ are degenerate in energy with the state ψ.

(e) Describe under which representations of $O(3)$ the following quantities transform: 1) an electric field $\vec{E}$; 2) a magnetic field $\vec{B}$; 3) $\vec{E} \cdot \vec{B}$; and 4) $\vec{E} \times \vec{B}$.

Exercise 3

Consider the special linear group $SL(2, \mathsf{R})$ of real 2×2 matrices with determinant 1 and its Lie algebra $sl(2, \mathsf{R})$.

(a) Give an explicit representation of the generators $a_i \in sl(2, \mathsf{R})$.

(b) Determine the dimension of $sl(2, \mathsf{R})$.

(c) Determine the center of $SL(2, \mathsf{R})$ (it is allowed to assume the defining rep is an irrep).

(d) Show whether the map ϕ from the general linear group $GL(2, \mathsf{R})$ into $SL(2, \mathsf{R})$, given by $\phi(A) = A/\sqrt{\det A}$, is a homomorphism or not.

(e) To which group is the factor group $GL(2, \mathsf{R})/SL(2, R)$ isomorphic?

(f) Show whether $SO(2)$ is an invariant subgroup of $SL(2, \mathsf{R})$ or not.